

Corrigendum

Corrigendum to “A tailored solver for bifurcation analysis of ocean-climate models” [J. Comput. Phys. 227 (2007) 654–679]

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This corrigendum contains a correction to some of the results in the article “A tailored solver for bifurcation analysis of ocean-climate models” [2], in which it was demonstrated how the specific mathematical structure of the equations governing oceanic flow can be exploited to build an efficient linear system solver for use in a continuation method. Compared to earlier work (e.g. [4]), an additional novelty was the implementation of the small slope approximation to Redi [3] isoneutral diffusion and Gent–McWilliams [1] (GM) stirring. These parameterizations provide a physically more realistic description of ocean mixing in z -coordinate models than horizontal diffusion. Unfortunately, the formulation of Redi and GM mixing in [2, Section 2.2.] contains several errors, which were carried over to the numerical implementation. New computations that used the correct formulation revealed qualitative differences with some of the results given in [2, Section 4.2.2.], in particular Fig. 7 and Fig. 8. Therefore, a correction of these errors will be provided below.

For completeness, we first correct the typographical errors that appeared in [2, Section 2.1.]. In the second term of the momentum equations [2, (4a,b)] a factor of r_0^{-1} , where r_0 is the Earth’s radius, is missing. Furthermore, the inclusion of the constant τ_0 in the last term is inappropriate, given that the wind-stress field (τ^ϕ, τ^θ) is defined to have units of Pa. In the definitions of the operators $L_u(u, v)$ and $L_v(u, v)$ a factor $\cos 2\theta$ was omitted in the numerator of the first metric term. They should read

$$L_u(u, v) = \nabla_H^2 u + \frac{u \cos 2\theta}{r_0^2 \cos^2 \theta} - \frac{2 \sin \theta}{r_0^2 \cos^2 \theta} \frac{\partial v}{\partial \phi}, \quad L_v(u, v) = \nabla_H^2 v + \frac{v \cos 2\theta}{r_0^2 \cos^2 \theta} + \frac{2 \sin \theta}{r_0^2 \cos^2 \theta} \frac{\partial u}{\partial \phi}.$$

Finally, the terms $R_T(T, S)$ and $R_S(T, S)$ in [2, (4e,f)] should be replaced by $-R_T(T, S)$ and $-R_S(T, S)$, respectively, since these are defined as the convergences of the diffusive tracer fluxes. All equations in [2, Section 2.1.] were coded correctly and hence these typographical errors have no implication for the correctness of the results shown in [2].

For the implementation of tracer mixing it is convenient to strictly discriminate between horizontal and isoneutral mixing. Therefore, the definition in [2, (6a,b)] of the mixing operator $R_C(T, S)$, where C may be either the temperature (T) or the salinity (S), is rewritten as

$$R_C(T, S) = \nabla_H \cdot \left([(1 - \eta_M)K_H + \eta_M K_I] \nabla_H C + (\eta_M K_I - \eta_C \kappa) \mathbf{S} \frac{\partial C}{\partial z} \right) + \frac{\partial}{\partial z} \left((\eta_M K_I + \eta_C \kappa) \mathbf{S} \cdot \nabla_H C + \eta_M K_I \mathbf{S} \cdot \mathbf{S} \frac{\partial C}{\partial z} \right) + \frac{\partial}{\partial z} \left(K_V \frac{\partial C}{\partial z} \right). \quad (1)$$

Here K_H, K_I and K_V are the horizontal, isoneutral and vertical diffusivity, and κ is the GM skew diffusive mixing coefficient. The two homotopy parameters η_M and η_C were introduced to facilitate smooth continuation between horizontal mixing ($\eta_M = 0$) and isoneutral mixing ($\eta_M = 1$), and between GM stirring disabled ($\eta_C = 0$) and enabled ($\eta_C = 1$). Specifically, when $\eta_M = \eta_C = 0$, tracer diffusion occurs only along geopotential directions with horizontal diffusion controlled by K_H ; when

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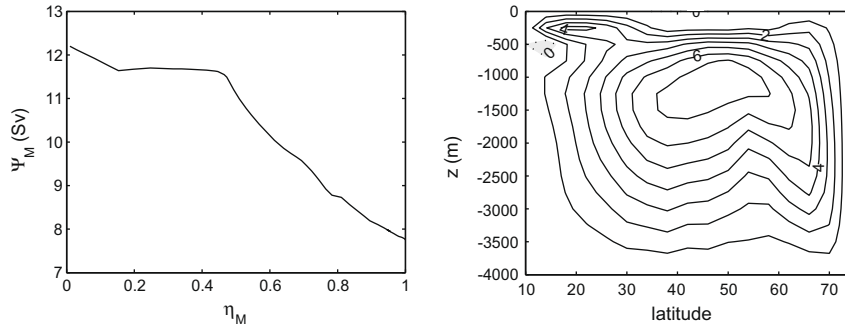


Fig. 1. (Left) Dependence of Ψ_M on η_M for $\eta_G = 0$. (Right) Pattern of the meridional overturning streamfunction for $\eta_M = 1$, $\eta_G = 0$; contours in Sv. These figures replace [2, Fig. 7a,b].

$\eta_M = 1$ and $\eta_G = 0$, diffusion is aligned with neutral directions with isoneutral diffusion controlled by K_I ; and when $\eta_G = 1$ the diffusion is augmented by GM stirring with mixing coefficient κ .

The definition of the neutral slope vector \mathbf{S} in [2, (7)] incorrectly appeared without a minus sign. It should read

$$\mathbf{S} = - \left(\begin{array}{c} -\alpha_T \nabla_H T + \alpha_S \nabla_H S \\ -\alpha_T \frac{\partial T}{\partial z} + \alpha_S \frac{\partial S}{\partial z} \end{array} \right). \tag{2}$$

Furthermore, application of the slope taper $f(x)$ given in [2, (8)] would inadvertently produce nonzero fluxes only in regions where isoneutral slopes are steep, whereas (1) is only valid for small slopes. In order to preserve numerical stability the taper should instead be defined as

$$f(x) = 1, \quad x \leq -\xi - \delta, \tag{3a}$$

$$f(x) = 1 - 3 \left(\frac{x + \xi + \delta}{\delta} \right)^2 + 2 \left(\frac{x + \xi + \delta}{\delta} \right)^3, \quad -\xi - \delta \leq x \leq -\xi, \tag{3b}$$

$$f(x) = 0, \quad x \geq -\xi, \tag{3c}$$

where $\xi = \|\nabla_H \rho\| (\tan \alpha_m)^{-1}$, $\delta = 0.05 \xi$, and α_m is the maximum permissible slope. The isoneutral and GM fluxes are then limited by multiplying K_I and κ in (1) with $f(\partial \rho / \partial z)$, i.e. $\{K_I, \kappa\} \rightarrow \{K_I, \kappa\} \times f(\partial \rho / \partial z)$.

In order to test the corrected code we consider the problem presented in [2, Section 4.2.]. We use a $16 \times 16 \times 16$ grid and start from the solution shown in Fig. 6a,b of [2] for which $\eta_M = \eta_G = 0$. When η_M is increased from 0 to 1 using $K_I = K_H$, while keeping $\eta_G = 0$, a 4.5 Sv ($1 \text{ Sv} = 10^6 \text{ m}^3 \text{ s}^{-1}$) decrease in the overturning strength (Ψ_M) results (Fig. 1 left panel), in contrast with the slight increase reported in [2]. The resulting pattern of the streamfunction (Fig. 1 right panel) also differs significantly from that in Fig. 7b in [2]. Since the isoneutral flux of density is zero by definition, the flow field should not depend on the strength of the isoneutral diffusivity (K_I). The solution shown here indeed respects this property. By the same reasoning, the velocity is insensitive to the shape of the taper. The dependence of Ψ_M on η_M is therefore invariant to the value of $\tan \alpha_m$, different from what is shown in Fig. 7a in [2]. Unlike what is claimed in [2] the product of the flawed implementation was never compared to results from the GFDL Modular Ocean Model (MOM). Such test was only conducted for the experiment with horizontal mixing (Fig. 6 of [2]). Yet, comparison of the new results with an equilibrium solution obtained with

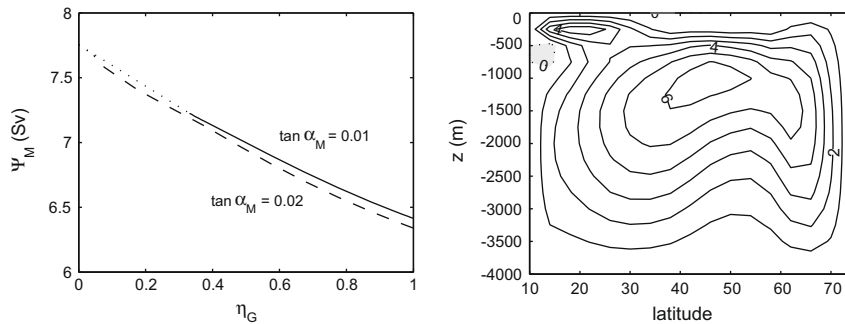


Fig. 2. (Left) Dependence of Ψ_M on η_G for $\eta_M = 1$; the drawn line is for $\tan \alpha_M = 0.01$; the dashed line for $\tan \alpha_M = 0.02$; and the dotted line is the part of the branches inferred from the available endpoints. (Right) Pattern of the meridional overturning streamfunction for $\eta_M = 1$, $\eta_G = 1$, $\tan \alpha_M = 0.01$; contours in Sv. These figures replace [2, Fig. 8a,b].

the MOM shows good agreement in the pattern of the meridional overturning streamfunction and a less than 0.5 Sv difference in its strength.

When starting from the above solution ($\eta_M = 1, \eta_G = 0$), continuation in η_G proves difficult due to slow convergence of the Newton–Raphson method. Instead, we prescribe $\eta_G = 1, \kappa = K_H$ and $\tan \alpha_M = 0.01$ and use implicit time stepping to obtain a steady state solution. The resulting meridional overturning streamfunction is shown in the right panel of Fig. 2. Rather than an increase in the overturning strength (as presented in Fig. 8a of [2]), activating GM stirring causes Ψ_M to decrease with 1.3 Sv. Comparison with a solution obtained with the MOM again results in a less than 0.5 Sv difference in overturning strength. Interestingly, continuation of steady states for decreasing values of η_G is feasible over a rather large range of η_G (Fig. 2 left panel). In addition, note the dependence of the shape of the branches on the maximum permissible slope angle (α_M) for this case.

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